NUMERICAL ANALYSIS

Assignment -2 (week 2)

Total Marks - 25

Posted on - 31/7/2017 (Monday);

To be submitted on or before-9/8/2017 (Wednesday), 23.59 hours.

Problems on

- Lagrange interpolation polynomial
- Divided difference interpolation polynomial
- error in interpolation

INSTRUCTIONS

- This is a question paper cum answer booklet.
- Take a print out of this.
- Present the details of the computations of the solution of each problem **which you will have to show** in the space provided at the bottom of the page.
- Fill in the answers in the space provided below each question.
- Scan the booklet and submit it as a pdf file before the deadline for evaluation.

1. Let $f(x) = lnx$. Given the table of values

x	0.4	0.5	0.7	0.8
$f(x)$	-0.916291	-0.693147	-0.356675	-0.223144

and the true value of $ln(0.6) = -0.510826$, find the Lagrange interpolation polynomial of degree 3 that interpolates $f(x)$ at $(x_i, f(x_i))$, $i = 0, 1, 2, 3$.
Fill in the answers in the blanks below.

(a)$L_0(0.6) =$ _____, (b)$L_1(0.6) =$ _____,

(c)$L_2(0.6) =$ _____, (d)$L_3(0.6) =$ _____,

(e) $ln(0.6) =$ _____,
(f) In the interval $(0.4, 0.8)$, if $A > E(0.6) > B$, then

$A =$ _____; $B =$ _____,
where $E(0.6)$ is $|ln(0.6) - p_3(0.6)|$. (8 marks)

Show your work for the solution of problem 1 in the space provided below.

2. Consider the interpolating polynomial $p_2(x)$ of degree at most 2 that interpolates f at the nodes $x_0 = 1$, $x_1 = 2$, $x_2 = 3$, where $p_2 = 3 + 4(x-1) + 1(x-1)(x-2)$. Obtain the first order divided difference $f[x_1, x_2]$ and also $f[3]$.
Fill in the blanks:
(a) The first order divided difference $f[x_1, x_2]$ is _____.

(b) $f[3] =$ _____. (3 marks)

Show your work for the solution of problem 2 in the space provided below.

3. The function e^x is tabulated at intervals of 0.01 between $x = 0$ and $x = 1$. Find an upper bound on the error (E) incurred by using linear interpolation in this table.
Fill in the blank:
$|E| \leq$ _____. (4 marks)

Show your work for the solution of problem 3 in the space provided below.

4. You are given the following information about $f(x)$

x	0	1	3	4
$f(x)$	2	1	0	1

(i) obtain Newton's divided difference interpolation polynomial that interpolates $f(x)$ at x_i, $i = 0, 1, 2, 3$.
(ii) Assume we know that the 4^{th} derivative satisfies $|f^{(4)}(x)| \leq 10$ for $x \in [0, 4]$. Find the estimate for $|E| = |f(2) - p_3(2)|$.
Fill in the blanks:
(a) If $f(x) \simeq p_3(x)$, then $p_3(2)$ is _____.

(b) $|E| \leq$ _____. (6 marks)

Show your work for the solution of problem 4 in the space provided below.

5. Use the divided-difference interpolation polynomial to estimate $f(3)$ from the following table

x	$x_0 = 0$	$x_1 = 1$	$x_2 = 2$	$x_3 = 4$	$x_4 = 5$	$x_5 = 6$
$f(x)$	1	14	15	5	6	19

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Fill in the blanks:
(a) $f(3) \simeq$ _____.

(b) $f[x_0, x_1, x_2] =$ _____, (4 marks)

Show your work for the solution of problem 5 in the space provided below.